THEORY OF MACHINE

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ENGINEERS INSTITUTE OF INDIA

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CONTENT

CHAPTER-1

MACHANISMS AND MACHINES

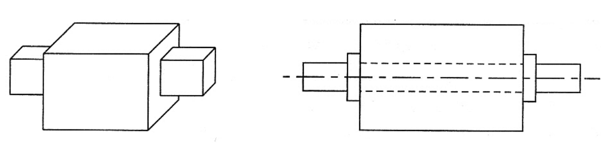
- **Mechanisms:** A mechanisms is a combination of rigid or restraining bodies so shaped and connected that they move up on each – other with a definite relative motion. A simple example of this is the slider crank mechanism used in an internal combustion or reciprocating air compressor.
- **Machine:** A machine is a collection of mechanisms which transmits force from the source of power to the resistance to be overcome and thus perform a mechanical work
- The study of a mechanism, involves its analysis as well as synthesis.
- **Analysis:** is the study of motions and forces concerning different parts of an existing mechanisms. In other words, it is the technique which allows the designer to critically examine an already existing or proposed design in order to judge its suitability for the task.
- **Synthesis:** is the process of contriving a scheme or a method of accomplishing a given purpose.
- The study of mechanisms can be divided into the following disciplines.

Kinematics: It deals with the relative motions of different parts of a mechanism without taking in to consideration the force producing the motions. Thus, it is the study from a geometric point of view, to know the displacement, velocity and acceleration of a part of a mechanism.

Dynamics: It involves the calculations of forces impressed upon different parts of a mechanism.

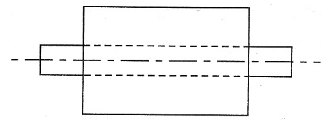
Type of constrained Motion:

(i) Completely constrained motion: when the motion between two elements of a pair is in a definite direction irrespective of the direction of the force applied, it is known as completely constrained motion.

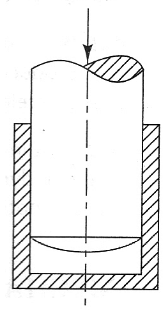


(a) (b)

(ii) Incompletely constrained motion: When the motion between two elements of a pair is possible in more than one direction and depends up on the direction of the force applied, it is known as incompletely constrained motion.



(iii) **Successfully constrained motion:** When the motion between two elements of a pair is possible in more than one direction, but is made to have motion only in one direction by using some external means, it is a successfully constrained motion.



Footstep bearing

Rigid And Resistant Bodies:

- A body is said to be rigid if under the action of forces, it does not suffer any distortion or the distance between any two points on it remains constant.
- Resistant bodies are those which are rigid for the purpose they have to serve.

Link:

- Link is defined as a resistant bodies or a group of resistant bodies with rigid connections preventing relative motion between them. The kinematic function of a link is to hold a fixed geometric relationship between the pair elements.
- A link is also known as kinematic link or element
- Links can be classified into binary, ternary and quaternary depending upon their ends on which revolute or turning pairs can be placed.

Kinematic Pair:

- A kinematic pair or simply a pair is a joint of two links having relative motion between them.
- Kinematic pairs can be classified according to:
  (i) Nature of contact
  (ii) Nature of mechanical constraint.
  (iii) Nature of relative motion.

(1) Nature of contact:

(a) Lower pair: A pair of links having surfaced or area contact between the members is known as a lower pair. The contact surfaces of the two links are similar.

Example: Nut turning on a screw, shaft rotating in a bearing, universal joint etc.

(b) High Pair: When a pair has a point or line contact between the links, it is known as a higher pair. The contact surfaces of the two links are dissimilar.

Example: Wheel rolling on a surface' cam and follower pair, ball and roller bearings.

➢ **Lower pair are mainly of six types:**

(i) Revolute pair

(ii) Prismatic pair

(iii) Screw pair

(iv) Cylindrical pair

(v) Spherical pair

(vi) Planar pair.

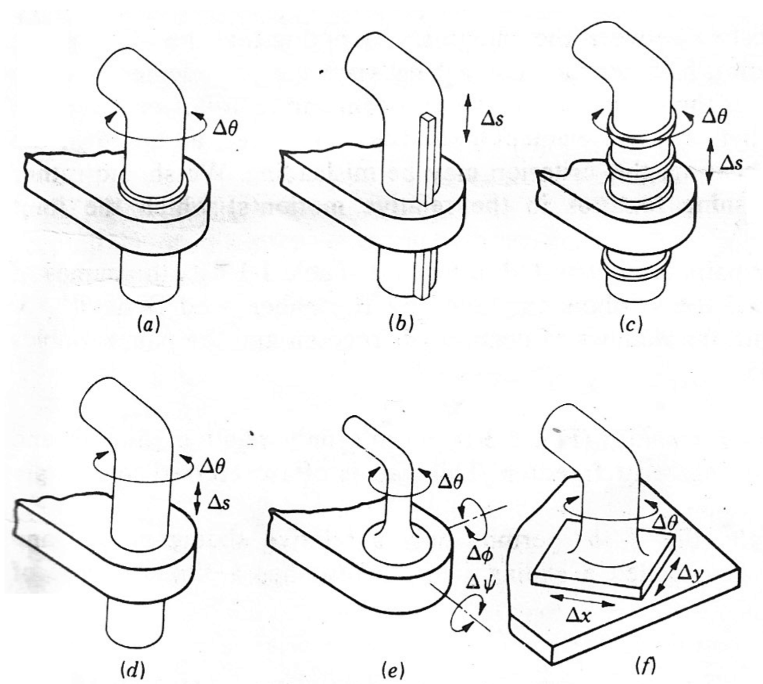


Figure: The six lower pairs: (a) revolute or pin, (b) Prism, (c) Helical, (d) Cylindric, (e) Spheric and (f) planar.

(i) Revolute Pair: This permits only relative rotation and hence has one degree of freedom. This pair is often referred to as a pin joint.

(ii) Prismatic Pair: This permits only a relative sliding motion and therefore is often called a sliding joint, it also has a single degree of freedom.

(iii) **Screw pair of helical Pair:** It has only one degree of freedom because the sliding and rotational motions are related by the helix angle of the thread. Thus, the pair may be chosen as either $\Delta s \text{ or } \Delta\theta,$ but not both. Note that the screw pair reverts to a revolute if the helix angle is mode zero and to a prismatic pair if the helix angle is made $90°$.

(iv) **Cylindrical Pair:** It permits both angular motion and an independent sliding motion. Thus, the cylindrical pair has two degree of freedom.

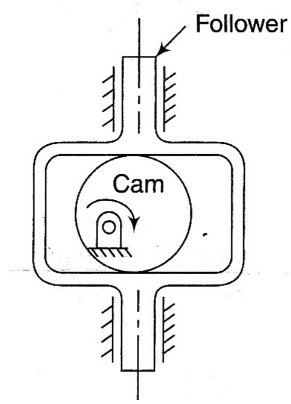
(v) **Globular or spherical Pair:** It is a ball – and – socket joint; it has 3 degree of freedom, a rotation about each of the co- ordinate axes.

(vi) **Flat Pair or planar Pair:** It is seldom, if ever, found in mechanisms in its undisguised form. It has 3 degree of freedom

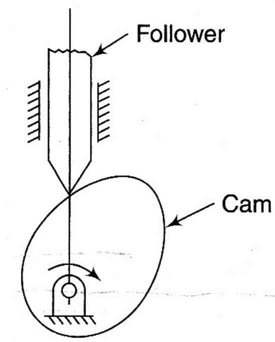
- Among the higher pairs, there is a subcategory known as wrapping pairs. Examples are the connection between a belt and pulley, between a chain and sprocket. In each case, one of the links has one – way rigidity.

(2) Nature of Mechanical Constraint:

(a) **Closed Pair:** When the elements of a pair are held together mechanically, it is known as a closed pair. All the lower pairs and some of the higher pairs are closed pairs.



(b) **Unclosed Pair:** When two links of a pair are in contact either due to force of gravity or some spring action, they constitute an unclosed pair. In this, the links are not held together mechanically.



(3) Nature of relative motion:

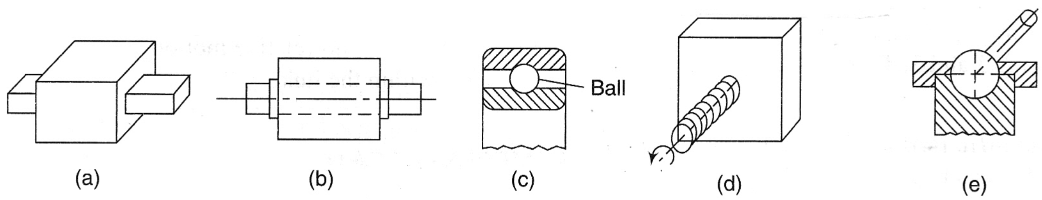


Figure: (a) Sliding Pair, (b) Turning pair, (c) Rolling pair, (d) Screw Pair, (e) Spherical pair

(a) Sliding Pair: If two links have a sliding motion relative to each – other, they form a sliding pair.

(b) Turning Pair: When one link has a turning or revolving motion relative to the other, they constitute a turning or revolving pair.

(c) Rolling Pair: When the links of a pair have a rolling motion relative to each – other, they form a rolling pair.

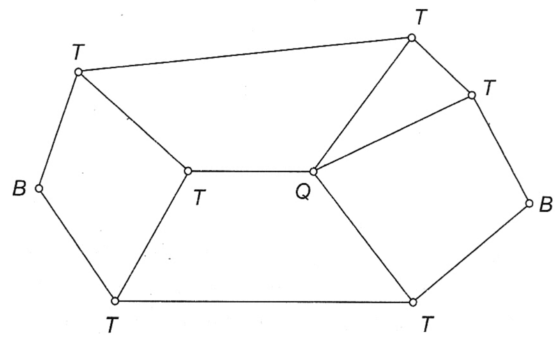
(d) Screw Pair: If two mating links have a turning as well as sliding motion between them, they form a screw pair.

(e) Spherical Pair: When one link in the form of a sphere turns inside a fixed link, it is a spherical pair.

Type of joints:

- The usual types of joints in a chain are
  - (i) Binary Joint.
  - (ii) Ternary Joint.
  - (iii) Quaternary Joint.

1. **Binary Joint:** If two links are joined at the same connection, it is called a binary joint.
2. **Ternary Joint:** If three links are joined at a connection it is known as a ternary joint. It is considered equivalent to two binary joints since fixing of any one link constitutes two binary joints with each of the other two links.
3. **Quaternary Joint:** If four links are joined at a connection, it is known as a Quaternary joint. It is considered equivalent to three binary joints since fixing of any one link constitutes three binary joints.



- In general, if 'n' numbers of links are connected at a joint, it is equivalent to (n-1) binary joints.

Degrees of Freedom:

- Degree of freedom of a pair is defined as the number of independent relative motions, both translational and rotational, a pair can have.
  Degree of freedom = 6 – (number of restraints).

Kinematic Chain:

- Kinematic chain is an assembly of links in which the relative motions of the links is possible and the motion of each relative to the other is definite.
- If the motion of a link results in indefinite motions of other links, it is a non – kinematic chain.
- A redundant chain does not allow any motion of a link relative to the other
- A linkage is obtained if one of the links of a kinematic chain is fixed to the ground.
- If motion of any of the movable links results in definite motion of the others, the linkage is known as a mechanism.
- If one of the links of a redundant chain is fixed, it is known as a structure or a locked system.
- The degree of freedom of a structure or a locked system is zero. A structure with negative degree of freedom is known as a superstructure.

Mobility of Mechanisms:

- The mobility of a mechanism is the number of input parameters (usually pair variables) which must be independently controlled in order to bring the device in to a particular position.
- Expressing the number of degrees of freedom of a linkage in terms of the number of links and the number of pair connections of different types in known as number synthesis.
- Let,

N	=	total number of links in a mechanism.
F	=	degrees of freedom.
P_1	=	No. of pairs having one degree of freedom.
P_2	=	No. of pairs having two degree of freedom, and so on.

In a mechanism, one link is fixed.

Therefore,

Number of movable links = (N – 1).

No. of degrees of freedom of (N-1) movable links = 6 (N-1). Each pair having one degree of freedom imposes 5 restraints on the mechanism, reducing its degrees of freedom by $5P_1$. Similarly, other pairs having 2, 3 and 4 degrees of freedom reduce the degrees, of freedom of the mechanism.

Thus,

$$F = 6 (N-1) – 5P_1 – 4P_2 - 3P_3 – 2P_4 – P_5.$$

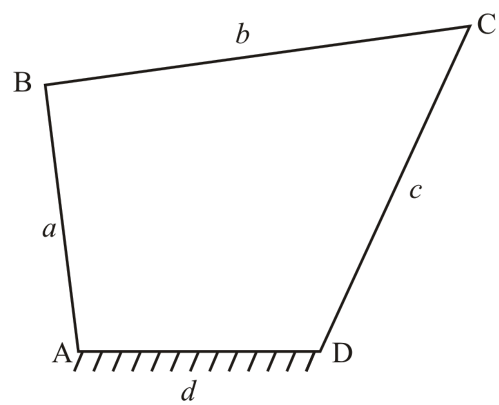
- For plane mecanisms,

$$F = 3 (N-1) – 2P_1 – P_2.$$

This is known as Gruebler's criterion for degrees of freedom of plane mechanisms in which each movable link possesses three degrees of freedom.

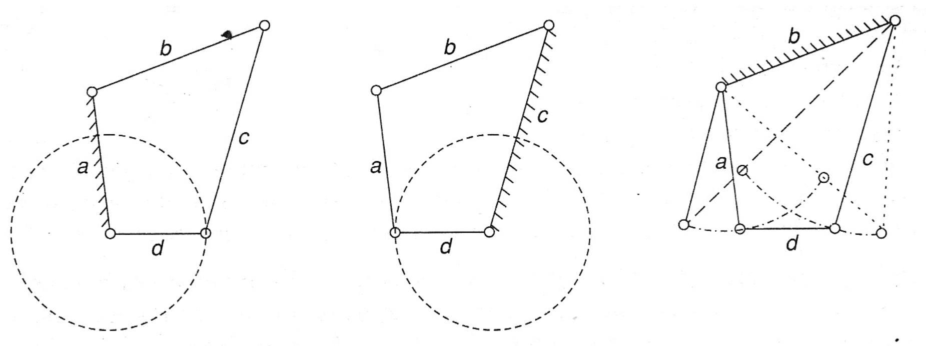
Four-Link Mechanism:

- A link that makes complete revolution is called the crank, the link opposite to the fixed link is called the coupler and the fourth link is called a lever or rocker if it oscillates or another crank, if it rotates.
- It is impossible to have a four – bar linkage if the length of one of the links is greater than the sum of the other three.



- The necessary conditions for the link 'a' to be a crank is
  - The shortest link is fixed.
  - The sum of the shortest and the longest link is less than the sum of the other two links
- If both the links 'a' and 'c' rotate through full circles, the link 'b' also makes one complete revolution relative to the fixed link 'd'

- The complete rotation of 'f' relative to 'a' is possible if the angle $\angle ABC$ can be more than 180^o from the figure, it is clear that these angles can not become more than 180^o. From the figure, it is clear that these angles can not become more than 180^o for the above stated condition
- As the relative motion between two adjacent links remains the same irrespective of which link is fixed to the frame, different mechanisms (known as inversions) obtained by fixing different links of this kind of chain will be as follows.



(a) (b) (c)

- If any of the adjacent links of link d, i.e., 'a' or 'c' is fixed, d can have a full revolution (crank) and the link opposite to it oscillates (rocks). In fig (a), 'a' is fixed, 'd' is the crank and 'b' oscillates whereas in fig. (b), 'C' is fixed, 'd' is the crank and 'b' oscillates. The mechanism is known as crank – rocker or crank – lever mechanism or rotary – oscillating converter.
- If the link opposite to the shortest link, i.e., link 'b' is fixed and the shortest link 'd' is made a coupler, the other tow link 'a' and 'c' would oscillates. The mechanism is known as a rocker – rocker or double rocker or double – lever mechanism or oscillating – oscillating converter.
- A linkage in which the sum of the lengths of the longest and the shortest links is less than the sum of the lengths of the other two links, is known as a class – I, four – bar linkage.
- When the sum of the lengths of the largest and the shortest links is more than the sum of the lengths of the other two links, the linkage is known as a class – II, four bar linkage. In such a linkage, fixing of any of the link always results in a rocker – rocker mechanism. In other words, the mechanism and its inversion give the same type of motion (of a double – rocker mechanism).
- **Grashof's Law:** A four – bar mechanism has at least one revolving link if the sum of the lengths of the largest and the shortest links is less than the sum of the lengths of the other two links.

Mechanical Advantage:

- The mechanical advantage of a mechanism is the ratio of the output force or torque to the input force or torque at any instant.

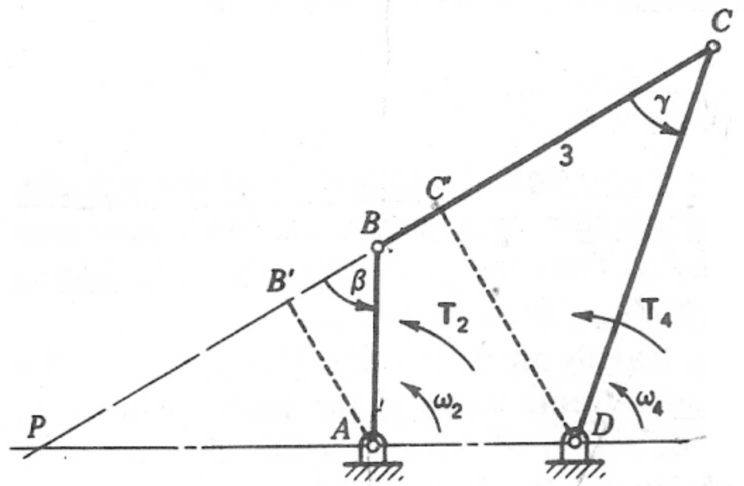


Figure: Four-bar linkage

Power input = Power output

$$T_2 \;\; \omega_2 \; = \; T_4\omega_4.$$

$$\Rightarrow MA = \frac{T_4}{T_2} = \frac{\omega_2}{\omega_4}.$$

- Mechanical advantage is the reciprocal of the velocity ratio. Either can be used as an index of merit in judging a mechanism's ability to transmit force or power.
- When the input link AB is in line with the copular BC and the angle γ between them is either zero or 180^o, it makes the mechanical advantage to be infinite at such positions. The extreme positions of the linkage are known as toggle position.

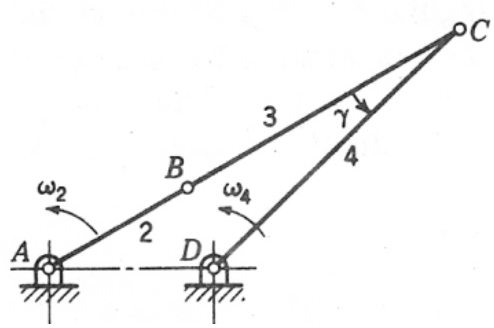
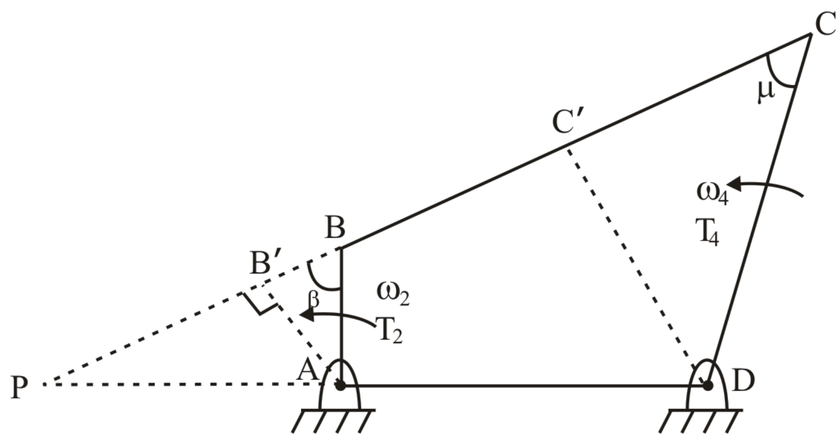


Figure: Four-bar linkage in toggle.

- In the case of crank – rocker mechanisms, the velocity ω_4 of the output link DC (rocker) become zero at the extreme positions.

Transmission Angle:

- The angle μ between the output link and couplar is known as transmission angle.
- For a particular value of force in the couplar rod, the torque transmitted to the output link (about the point D) is maximum when the transmission angle μ is 90^o. If links BC and DC become coincident, the transmission angle is zero and the mechanism would lock or jam.
- If μ deviates significantly from 90°, the torque on the output link decreases. The best mechanisms, therefore, have a transmission angle that does not deviate much from 90^o
- Transmission angle is also often used as an index of merit for a four – bar linkage.

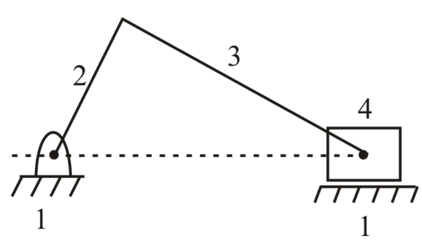


$$\frac{T_4}{T_2} = \frac{\omega_2}{\omega_4} = \frac{R_{CD}\sin\mu}{R_{BA}\sin\beta}$$

The Slider-Crank Chain:

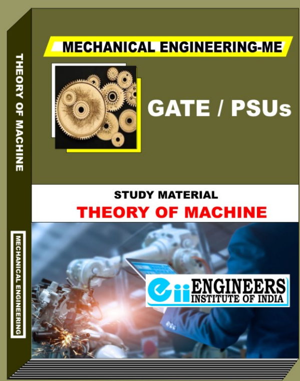
First Inversion:

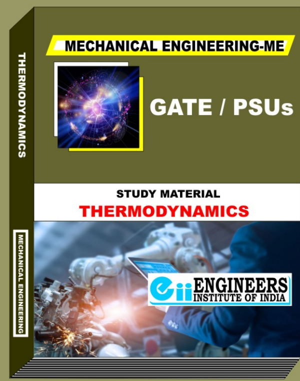
- This inversion is obtained when link 1 is fixed and links 2 and 4 are made the crank and the slider respectively.

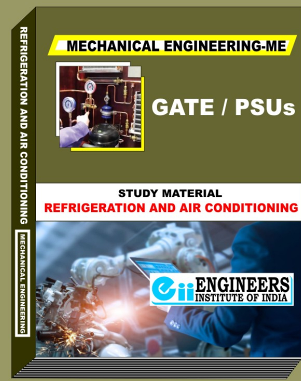


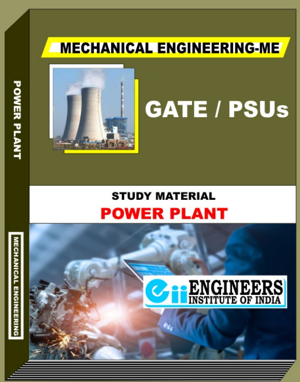
- Its main application is in Reciprocating engine, Reciprocating compressor.

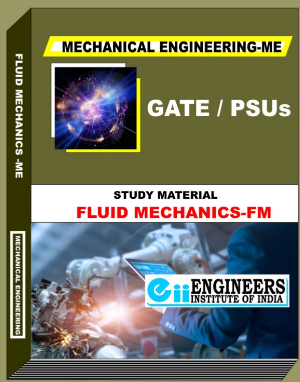
Published Books

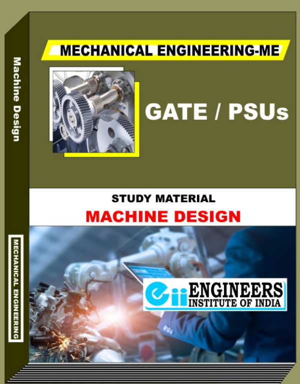


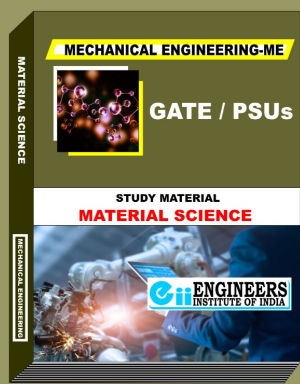


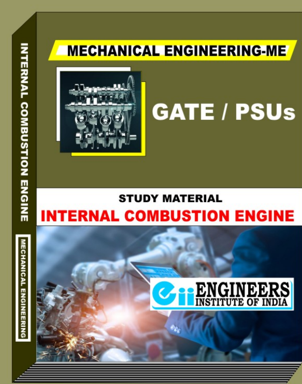












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